have the teaching of mathematics remain stagnant. All articles that have thus far been written in connection with the work show that the members of the Commission desire the same substantial progress in teaching mathematics that is generally found in all other subjects, and no one has taken the attitude of opposition to all that exists, nor has anyone appeared with the dictum that all that is, is bad and that the new and untried is always good. This general attitude of the Commission, if we may judge by what has thus far been done, will argue for lack of progress with some earnest workers, but it is probable that it will meet with the approval of the substantial leaders in mathematics and in education today.

In the second place, it is evidently not the purpose of the Commission to make any direct attempt at uniformity in the various countries. The very manner of conducting the investigation shows that each country has gone to work at the problem in the way that seems to it best, without the slightest regard to the methods employed in other countries; and this independence, this recognition of the peculiar needs, aims, and means at the disposal of the various countries, will undoubtedly continue to be seen as the work progresses. It is not, therefore, proposed to revolutionize the teaching of mathematics, nor is it proposed to attempt to bring all nations to one point of view as to schools, as to teaching, as to subjects taught, or as to curricula.

From the time of the Greek mathematicians until now, it has been recognized that a negative statement is never satisfactory, and so it is proper to proceed to a positive statement of some of the reasons for this word-wide investigation.

The prime reason is unquestionably to be found in the value of an interchange of views and experiences among nations. Why should the American Association for the Advancement of Science, or the American Mathematical Society, or the American Federation of Teachers of the Mathematical and Natural Sciences exist? Is it not enough that we have the faculty meetings, and local reading circles, and even state organizations? Have we not journals and reports and text-books and treatises to keep us mentally alert? Then why do we need such national gatherings as the present one, and why should we give up our well-earned holidays to attend them? These questions seem puerile when applied to our present gathering, for each of us is conscious that he gains much inspiration from such a meeting, and that he returns to his work with a broader horizon because he has been conducted to other points of view. He sees the national problem instead of his narrow local one, and his work is the stronger because of this fact. You cannot weigh this value, nor is it to be measured in volume or in depth, for it is incommensurable with any units of the laboratory or of trade,—but our sixth sense knows it, and it is even more real than it would be if expressed in the mere figures of a scale.

And if the village point of view is narrow, and even that of the city,

and of the state, and of this section or that of a country like ours, so too is the national point of view narrow as compared with that of the world of which it must be, whether it will or no, a part. This, then, is the great *raison d'être* of the Commission,—to act as a clearing house, to let each part of the world see what the other part is doing in the teaching of mathematics. Uniformity is no more desirable nor attainable than uniformity in dress, in government, or in speech. The movement will tend in this direction, just as all intercourse tends to uniformity in these other lines, but centuries will elapse before any of these results will be reached, if unfortunately they are ever attained.

Such a movement, looking to telling the rest of the world what any specific nation is doing in the teaching of mathematics, carries with it the necessity, on the part of that nation, of taking stock of its own results, and of indulging in a very healthy form of introspection. It is most interesting to those of us who are intimately connected with the investigation, to see how much we all have to learn of the courses of study, the amount that can be well done in a year, and the topics that are treated in the various parts of our country. I will illustrate by two simple but typical questions: (1) What are the aims, and consequently what should be the nature of the work, in mathematics, in the large city commercial high schools of this country? (2) What are the requirements in mathematics for the doctor's degree in say three universities belonging to the Association of American Universities? It would seem that these questions ought to be easily answered by several persons in an audience like this, but we find difficulty in discovering anyone to answer them, in a much larger circle than the one here represented. A second large reason, then, for this investigation is that we may know not only what the world at large is doing, but also our own aims and curricula and efforts at improvement.

But more specfically, what is it that we propose to investigate?—that the rest of the world wishes to know? that we ourselves need also to inquire into?

In the first place, it was at once seen that the limitation to secondary schools was meaningless. The term signifies one thing in America, but something radically different in Germany, and something different still in other countries. Even with us there is no well-marked boundary between the elementary school with six, seven, eight, or nine years, and the secondary school with six, five, or four years, or even less. It was therefore early decided to extend the investigation so that it might include the entire field of mathematics required for any purpose, from the kindergarten through the college, the technical school, and the professional school. Besides this, since the investigation includes the preparation of teachers, it must also include a statement of the general nature of the graduate work in mathematics in the universities, and in particular of the requirements for the various advanced degrees in universities of recognized standing.

With the general scope of the work thus set forth, I pass to a consideration of a few of the large topics to be investigated in this country.

The first of these topics is the mathematics of the general elementary school, where a few years ago the obsolete business problem and the remains of the ancient theory of numbers were looked upon as of paramount importance, but where now the needs of modern daily life make up the bulk of the work. The change here has been greater than in any other part of our American education in mathematics, save only in the graduate university work. To report upon this change, and to set forth the present status of the work, for the various types of schools, is a labor of no little difficulty, and the world at large will look upon it as one of our most important contributions.

There is next considered certain special kinds of elementary schools, of which we hear and know little because they are still so new. The mathematics of the trade school is not that of the public school, nor is that of the large corporation industrial schools, now springing up in various parts of the country, like that of the trade school. We have not yet differentiated in this field as they have, for example, in Switzerland, and perhaps we shall never need to do so, but it is certain that these schools are with us, and that others are coming, and that their needs in mathematics, and their ways of meeting these needs, will be of interest to all who are engaged in the furthering of education.

One of the largest problems that confronts us is the study of the public general secondary schools of the country, partly because it is here that mathematics, beyond mere computation, may be said to begin, and partly because there is more criticism of this work than of any other save in the first two years of college. Fortunately we have in Mr. Evans of this city an organizer of proved ability, and with him we have a committee of very earnest workers. Anyone who is at all familiar with these public schools knows that there is much divergence in the courses in mathematics in different parts of the country, and in schools intended for boys, for girls, or for both. It is also apparent that the new movement towards a six-year elementary school, followed by a six-year high school, will considerably modify the mathematics of the latter. With six years in the high school, the present mathematics will begin earlier, it may possibly be taken more leisurely, and eventually it may well lead into higher lines of work. The problem, therefore, assumes an interest that a few years ago would not have been expected. Add to this the numerous efforts made at present to frame new syllabi in subjects like algebra and geometry, and it is evident that the Committee has a labor of no small extent to perform.

Then there is another problem that is quite different, namely, that of the private secondary school, whether for boys alone, for girls alone, or for both together. These are generally fitting schools for college. Instead of a small per cent. going to college, as in the public schools, the rest going

into business, here these per cents are reversed, and the nature of the work in mathematics is, or may be, very different. As a matter of fact, it is different, and more different than one would at first expect.

An important question, and one that is by no means settled, is that which relates to the mathematics in the normal schools, public and private. Upon this depends the equipment of the teachers who are to instruct the new generation of elementary pupils in the early principles of mathematics, largely determining whether or not they will care for the subject in later years. Of late there has been a tendency to weaken the mathemathics in these schools, and it is hoped that this report will determine the needs and perhaps the future policy of such institutions in this important field.

There is arising in our country at present a type of secondary school that is well known on the continent of Europe but that has yet to make its way with us. This is the technical secondary school. We have, for example, even in the present year a notable development of the agricultural high school, of the commercial high school, and of the general technical or manual training high school. What shall the mathematics be in these institutions? Shall it be Euclid revised and algebra through quadratics, as in the ordinary high school? Or shall it be mathematics correlating directly with the peculiar work of the type of school in question?

Related to these schools is the rapidly increasing number of evening technical schools, of private correspondence schools, of schools for licensed accountants, and of industrial schools for negroes and Indians. Some of these institutions are teaching mathematics even through applied calculus, and are doing it with a spirit that would put the average college class to shame, and it will help us all to know their work more fully.

One of the most important problems that we have relates to the examination question. We commonly look at England as examination-ridden, and in the East we often hear it said that the West and Middle West teach mathematics so loosely that they dare not submit their pupils to the Eastern tests. Here, therefore, are these types,—the English, the extreme of the examination system; the West, the extreme of the non-examination system; and the East occupying a middle ground. It is well that we endeavor fairly to weigh the advantages of these various plans, to consider the nature of the promotion in the elementary grades, to investigate the results of the several plans for admission to college, and to inform ourselves as to the examinations in mathematics set by the state and local authorities for those desiring to teach in the public schools. If we do not, as a result of all this work, attain to uniformity, at least we shall be more tolerant of the plans of others, if not of our own.

America has of late branched out geographically for good or for ill, and it is expected that we shall hear, as part of this report, of the work being done in mathemathics in the Philippines, in Hawaii, in Porto Rico, and in Alaska. Nor will this be merely a discussion of the elementary schools,

for Hawaii has more than one good college, and Manila has a university so old that most of ours seem young in comparison.

We have in this country certain influences that tend to improve the work of the teacher that will be of interest to other countries, and about which we ourselves may profitably be better informed. Our scientific societies and periodicals are stimuli to our scientific activity, and are characterized by a generosity of spirit that will be the envy of several countries. The nature of these organizations and publications, including the American Association for the Advancement of Sciences, and our various mathematical journals, will be the subject of one of our important reports. We are favored, too, in having so many associations of teachers of mathematics, and to them we owe most of the syllabi in algebra and geometry that are now appearing. The teachers' institute, as conducted in our various states, seems to be indigenous to our country, and while it has not in general done much for mathematics, it has, in the high school sections, often proved of service. These and similar questions will be discussed by a committee appointed for this special purpose.

There has grown up in this country a line of technological work which we have reason to regard with much satisfaction,—that of collegiate grade, represented by institutions like the Massachusetts Institute of Technology. Such schools are not unique with this country; indeed, we borrowed the idea from Germany, Switzerland, France, and Belgium. But they have steadily increased in number, in dignity, and in their ability to offer strong courses in engineering mathematics. But what this work is, and how it differs in the technological schools from the work in the technological departments of colleges and universities, are matters upon which we should all welcome exact information.

Allied to this problem is that of the mathematics taught in other professional schools, such as those for the training of army and navy officers, and particularly in the higher institutions for the training of teachers that are beginning to be established in this country. The question of how we proceed to train a teacher of mathematics for the secondary school and college, in the highest type of college for this purpose, will be of interest abroad and not without its value at home.

From the educational standpoint the mathematical work of the college is commonly felt, in this country, to be the least progressive. Perhaps it should be so, but at any rate it will be of service to us to know exactly what is expected of the freshman year, and of the sophomore year, in an average American college. Whatever exact information we can get as to the aims, the general plan of presentation, and the results, of these two years, will be stimulating in all institutions of this type. Perhaps as important a question as we have to consider is that relating to the nature of the mathematics in the women's colleges, compared with that in the colleges for men. Is it the same? Is the ability the same? Such are some of the questions that the world of mathematics would like to have scientifically considered.

Perhaps the most interesting question of all, from the standpoint of mathematics, is that relating to the graduate work done in the universities of this country,—that is, in the few institutions of high grade that are generally accepted as prepared for genuine work of this character. One of our committees has this problem in hand, one of its sub-committees having the question of the courses of instruction, another of the preparation for research and the doctor's degree, and another of the preparation of instructors for colleges and universities. The report of this committee may well be expected to set a standard of excellence that shall be recognized for many years to come, in this country.

And finally, there are certain general questions that need to be considered, particularly for the information of other countries. One of these has to do with a schematic survey of American educational institutions, their sequence and inter-relations. The fact that this particular topic is under the direction of a sub-committee headed by the Commissioner of Education of the State of Massachusetts, is a guarantee that the work will be well done. The second of these general topics has to do with the scope and arrangement of the mathematical curriculum as a whole, as distinguished from the arrangement of limited portions as considered by various other committees.

In America, as in some other countries, the work is carried on by means of committees. Certain other countries are conducting the investigation through individuals. In this country we have fifteen committees, sixty-one sub-committees, and a total of about two hundred and seventy members. These committees and sub-committees are working out their problems in their own way, some by means of elaborate questionnaires, others through associations, others from statistical reports, and others through investigations previously made by their members. It is hoped that a considerable number of the sub-committee reports will be ready in February, 1910, and that all of the committee reports will be ready the following May. Some extensions must be allowed, but it is hoped that these will not delay the ultimate stages of the work. The American report must be ready a year hence, and the great labor that must be expended upon it leads to the hope that all who receive questionnaires will respond heartily and speedily to the end that the burden may not be made any heavier.

Such is the general purpose and such is the general plan of this work. It imposes upon all of us who are connected with it a very heavy burden. But the work must be done for the reputation of the country and for the sake of its effect both here and abroad. It is to the sympathetic interest of societies like those that I address that we look for our chief encouragement and assistance, and it is upon teachers and mathematicians like yourselves that we have called to fill the various committees and to work with us, *con amore* as all of us must, to make the labor result in success and be of value to ourselves and to our confères in other parts of the world.

NOTE ON MAXIMA AND MINIMA BY ALGEBRAIC METHODS.

By N. J. LENNES, Massachusetts Institute of Technology.

The pedagogical interest of this note seems sufficient to warrant its publication. The problems on maxima and minima in the Calculus are admittedly of considerable value in that they furnish interesting and concrete applications of the theory of derivatives, and also because of the importance of the problems themselves. Since nearly all problems of this class, which involve algebraic functions only and which are readily soluable by finding values which make the first derivatives vanish, may also be solved very easily by elementary algebraic methods, it would seem desirable to do so. And all the more so since the algebraic method affords an excellent illustration of the use of the graph and besides brings into play the relations between the roots and the coefficients of an algebraic equation. To those who never reach the calculus but who do study college algebra it may be worth while, for the sake of these problems themselves, to solve them rather than an equal number of abstract exercises.

If the expression whose maximum or minimum we seek is of the second degree we need only to find the highest or lowest point of a curve

$$y=a_0x^2+a_1x+a_2$$

which may be done by solving this equation simultaneously with

$$y=b$$

and then giving b suc value that the two roots are identical. That is, the line is made tangent to the curve.

If in a similar manner we seek the intersection points of the cubic

$$y=a_0x^3+a_1x^2+a_2x+a_3$$

and the line

$$y=b,$$

we have an equation yielding three roots. If the line is to be tangent to the curve at one point two of these three roots are coincident. Representing the coincident roots by r_1 and the remaining root by r_2 we have

$$\begin{cases} 2r_1+r_2=\dfrac{a_1}{a_0} \\ 2r_1r_2+r_1^2=\dfrac{a_2}{a_0} \end{cases}.$$

Of the two values of r_1 obtained by solving this set of equations, one clearly corresponds to a maximum and the other to a minimum. Indeed, this is at once evident to the eye by constructing a graph of the cubic.

In case a_0 is positive the maximum of the function $a_0x^3+a_1x^2+a_2x+a_3$ is to the *left* of its minimum, and hence the smaller value of r_1 obtained above corresponds to a maximum and the greater value to a minimum.

If $r_1=r_2$ or if these are both complex there is, of course, neither maximum nor minimum.

Clearly, the method applies to any curve whose equation is reducible to the form

$$x^3+a_1x^2+a_2x+f(y)=0.$$

where a_1 and a_2 are constants and $f(y)$ any real function whatever of y. It may, of course, be impossible to find the value of y at the maximum or minimum point, though the values of x for such point may always be found.

In a similar manner the maxima and minima points of the curve

$$x^n+a_1x^{n-1}+\ldots+a_{n-1}x+f(y)=0$$

may be found by solving an equation of the $(n-1)$st degree. In the case of the biquadratic the details are as follows:

Suppose two of the roots of the equation

$$x^4+a_1x^3+a_2x^2+a_3x+f(y)=b$$

are coincident. Denote the coincident roots by r_1 and the other two roots by r_2 and r_3. Then

$$\begin{cases} 2r_1+r_2+r_3=-a_1 & (1)\\ r_1^2+2r_1r_2+2r_1r_3+r_2r_3=a_2 & (2)\\ r_1^2r_2+r_1^2r_3+2r_1r_2r_3=-a_3 & (3)\end{cases}$$

From (1) and (2),

$$r_1^2+2r_1(-a_1-2r_1)+r_2r_3=a_2 \qquad (4)$$

From (1), (4) and (3),

$$r_1^2(-a_1-2r_1)+2r_1[a_2-r_1^2+2r_1(a_1+2r_1)]=-a_3 \qquad (5)$$

or

$$4r_1^3+3a_1r_1^2+2a_2r_1=-a_3 \qquad (6)$$

The three values of r_1 obtained by solving equation (6) will be the values of x for which the given curve has turning points. The middle value will necessarily belong to a maximum and the other two to minima. In case two of the roots are coincident the remaining root will give a minimum and in case they are all coincident their value gives a minimum.

Again, these statements are at once rendered evident by means of the graph.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

NOTE.—Thomas S. Clarkson remarks that the solution of 321 does not satisfy the conditions of the problem, since the whole number of shares is not to exceed 200. He contends that the problem is impossible as stated, and suggests that it may have been intended that each man's share is not to exceed 200. We think the problem is stated as intended and is therefore impossible. ED. F.

325. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

I have a chronometer whose rate is uniform. When it indicates t_1 time at Washington I find that it is h_1 hours slow. I take it to Philadelphia and when it indicates t_2 time, the local time of Philadelphia is h_2 hours faster. I bring my chronometer back to Washington and find that when it indicates t_3 time it is h_3 hours slow. If $t_1 = 5$ A. M., $t_2 = 7$ hours. 54 minutes, $t_3 = 11$ hours, 46 minutes A. M., $h_1 = 1$ hour, $h_2 = 1\ 203/900$ hours, $h_3 = 1\ 7/30$ hours, find the difference of longitude between Washington and Philadelphia.

Solution by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill. and the PROPOSER.

$(h_3-h_1)/(t_3-t_1)$=error for one hour.

$(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=error at time t_2 in Philadelphia.

$\therefore h_1+t_2+(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=time at Washington.

h_2+t_2=time at Philadelphia.

$h_2+t_2-h_1-t_2-(t_2-t_1)(h_3-h_1)/(t_3-t_1)$=difference of time between the two cities=T.

$$\therefore T=\frac{h_2(t_3-t_1)+h_3(t_1-t_2)+h_1(t_2-t_3)}{t_3-t_1}.$$

$15T=L$=difference in longitude.

$$\therefore L=\frac{15(h_1t_2+h_2t_3+h_3t_1-h_1t_3-h_2t_1-h_3t_2)}{t_3-t_1}.$$

Putting h_1=1 hour, $h_2=1\frac{203}{900}$ hours, $h_3=1\frac{7}{30}$ hours, t_1=5 hours, t_2= 7 hours 54 minutes, t_3=11 hours 46 minutes, we get

$$L=\tfrac{11}{60}3^\circ=1^\circ\ 53'.$$

Also solved by T. S. Clarkson.

326. Proposed by R. D. CARMICHAEL, Princeton University.

Is the series, of which the nth term is $\dfrac{1.3.5.7\ldots(2n-1)}{(n+1)!\,2^n.(2n+3)}$ convergent? If so, find its sum.

Solution by E. B. ESCOTT, Ann Arbor, Mich.

Apply the test given in Hall and Knight's *Higher Algebra*, 3rd ed., p. 244.

If $\underset{n=\infty}{\text{Lim}}\left[n\left(\frac{u_n}{u_{n+1}}-1\right)\right]>1$, the series is convergent. In this case

$$\frac{u_n}{u_{n+1}}=\frac{(n+2)2(2n+5)}{(2n+1)(2n+3)}.$$

The above limit$=\frac{5}{2}$. Therefore, the series is convergent.

To find its value, we may use the following method due to Euler:

$$\text{Let } y=\ldots+\frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^n\,(2n+3)}x^{2n+3}+\frac{1.3.5\ldots(2n+1)}{(n+2)!\,2^{n+1}(2n+5)}x^{2n+5}+\ldots$$

$$\frac{dy}{dx}=\ldots+\frac{1.3.5\ldots(2n-1)}{(n+1)!2^n}x^{2n+2}+\frac{1.3.5\ldots(2n+1)}{(n+2)!2^{n+1}}x^{2n+4}+\ldots$$

$$\frac{d^2y}{dx^2}=\ldots+\frac{1.3.5\ldots(2n-1)}{n!2^{n-1}}x^{2n+1}+\frac{1.3.5\ldots(2n+1)}{(n+1)!2^n}x^{2n+3}+\ldots$$

Divide by x^3 and integrate,

$$\int\frac{1}{x^3}\frac{d^2y}{dx^2}dx=\ldots+\frac{1.3.5\ldots(2n-3)}{n!\;2^{n-1}}x^{2n-1}+\frac{1.3.5\ldots(2n-1)}{(n+1)!2^n}x^{2n+1}+\ldots$$

Multiply by x and integrating again,

$$\int x\int\frac{1}{x^3}\frac{d^2y}{dx^2}dx\,dx=\ldots+\frac{1.3.5\ldots(2n-3)}{n!\;2^{n-1}(2n+1)}x^{2n+1}+\frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^n\,(2n+3)}x^{2n+3}$$

$$+\ldots=y.$$

Then differentiating, dividing by x, and differentiating again, we have

$$\frac{1}{x^3}\frac{d^2y}{dx^2}=\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^2}\frac{dy}{dx}.$$

This differential equation may be written

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0.$$

Solving the differential equation, we have

$$y=\frac{c}{2}[x\sqrt{(1-x^2)}+\sin^{-1}x].$$

To verify that this gives the given series, we have by differentiating,

$$\frac{dy}{dx}=c\sqrt{(1-x^2)}=c(1-\frac{1}{2}x^2-\frac{1}{2!\,2^2}x^4-\frac{1.3}{3!\,2^3}x^6-\ldots$$

$$-\frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^{n+1}}x^{2n+2}-\ldots).$$

Integrating, $y=c(x-\frac{1}{2.3}x^3-\frac{1}{2!2^2.5}x^5-\frac{1.3}{3!2^3.7}x^7-\ldots$

$$-\frac{1.3.5\ldots(2n-1)}{(n+1)!\,2^{n+1}(2n+3)}x^{2n+3}-\ldots).$$

Putting $c=-2$, we have

$$\lim_{x=1}[-(x\sqrt{(1-x^2)}+\sin^{-1}x)]=-\frac{\pi}{2}=-2+\frac{1}{3}+\frac{1}{2!\,2.5}+\frac{1.3}{3!2^2.7}+\ldots$$

Also solved by G. B. M. Zerr, who found for the limit of the sum of n terms of the series as $n=\infty$, $\S-\frac{1}{2}\pi$.

GEOMETRY.

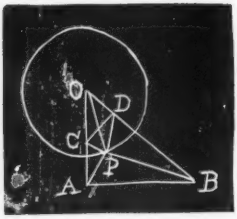
349. Proposed by J. A. CAPARO, Notre Dame University, Notre Dame, Indiana.

Given the radius of a circular smooth cylinder and its position with respect to a source of light and the eye. Find a geometrical construction to determine the line of brilliancy.

Solution by C. N. SCHMALL, New York City.

By a simple examination of the data* it is clear that the problem reduces to the following: Given a circle and two points A, B, outside (in same plane), to find a point P on the circumference, such that AP and

*See *Encyclopedia Britannica*, Vol. XIV, page 589.



BP make equal angles with the radius drawn to P.

This is known as Alhazen's Problem and does not admit of a solution by ruler and compasses only. However, by the use of an hyperbola an approximate solution can be effected.

Construction. Let O be the center of the circle, a its radius. Take C and D so that $AO.OC = a^2 = BO.OD$.

Now, the locus of the vertices of the triangles whose base is CD and whose base angles have a constant difference $(OCD - ODC)$ is well known to be a hyperbola. This will cut the circle in four points, of which let P be one. This is the point required.

Proof. $\angle CDP - \angle DCP = \angle OCD - \angle ODC$ (by construction). Transposing and adding,

$\therefore \angle OCP = \angle ODP \ldots (1)$.

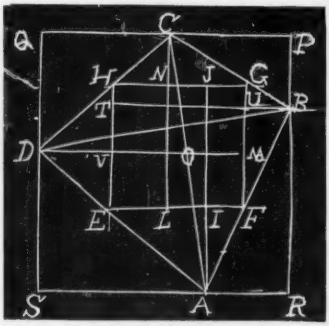
Also in the triangles AOP, POC, we have $AO:OP = OP:OC$ (by construction).

$\therefore \angle APO = \angle OCP$. Similarly, $\angle BPO = \angle ODP$ by (1).

$\therefore \angle APO = \angle BPO$, and $\therefore \angle APR = \angle BPR$. Q. E. F. Q. E. D.

350. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Given the quadrilateral $AB=a=225$, $BC=b=153$, $CD=c=207$, $DA=d=135$, $AC=e=240$. Find the side of the square inscribed in this quadrilateral having a corner in each side.



Solution by the PROPOSER.

Let $ABCD$ be the given quadrilateral, $EFGH$ the inscribed square, $RPQS$ the circumscribed rectangle having its sides parallel to the sides of the square. Draw AIJ, BUT, CNL, DVM perpendicular, respectively, to EF and HG, FG and HE, HG and EF, HE and GF.

Let $AB=a$, $BC=b$, $CD=c$, $DA=d$, $AC=e$, $BD=f$, O the intersection of AC, BD, $\angle COB=\beta$, $\angle ACL=\theta=\angle CAJ$, $\angle BDM=\phi=\angle DBT$, $\angle BAC=\delta$, $\angle DAC=\gamma$, $\angle BCA=\rho$, $\angle DCA=\mu$, area $ABCD=\triangle$.

Then $\phi=\frac{1}{2}\pi-(\beta-\theta)$, $RB=a\cos(\delta-\theta)$, $RA=a\sin(\delta-\theta)$, $DS=d\cos(\gamma+\theta)$, $AS=d\sin(\gamma+\theta)$, $DQ=c\cos(\mu-\theta)$, $CQ=c\sin(\mu-\theta)$, $BP=b\cos(\rho+\theta)$, $PC=b\sin(\rho+\theta)$, $AI+x+CN=AJ+CN=e\cos\theta$, $BU+x+DV=BT+DV=f\cos\phi$.

$\therefore BT+DV=f\sin(\beta-\theta)$.

$x^2+\frac{1}{2}x(AI+BU+CN+DV)=\triangle=\frac{1}{2}x(AI+x+CN)+\frac{1}{2}x(BU+x+DV)$.

$\therefore x[e\cos\theta+f\sin(\beta-\theta)]=2\triangle \ldots (1)$.

$$\triangle+\tfrac{1}{4}[a^2\sin 2(\delta-\theta)+b^2\sin 2(\rho+\theta)+c^2\sin 2(\mu-\theta)+d^2\sin 2(\gamma+\theta)$$
$$=ef\cos\theta\sin(\beta-\theta)\ldots(2).$$

Reducing (2), and remembering that $4\triangle-2ef\sin\beta=0$, we get

$$\tan 2\theta=\frac{a^2\sin 2\delta+b^2\sin 2\rho+c^2\sin 2\mu+d^2\sin 2\gamma-2ef\sin\beta}{a^2\cos 2\delta-b^2\cos 2\rho+c^2\cos 2\mu-d^2\cos 2\gamma-2ef\cos\beta}\ldots(3).$$

When $a=225$, $b=153$, $c=207$, $d=135$, $e=240$. Then $f=277.4$, $\triangle=30656.46$, $\delta=38^\circ\ 14'\ 54''$, $\rho=65^\circ\ 33'\ 40''$, $\gamma=59^\circ\ 24'\ 36''$, $\mu=34^\circ\ 9'\ 22''$, $\beta=67^\circ\ 3'\ 52''$.

$\therefore\tan 2\theta=-.345608=2\tan\theta/(1-\tan^2\theta)$.

$\therefore\tan\theta=5.954833$ or -0.167930.

$\theta=80^\circ\ 28'\ 2''$ or $170^\circ\ 28'\ 2''$ and $x=-2496.97$ or -121.044.

Therefore, two squares can be inscribed in this quadrilateral, the smaller one truly inscribed and the larger with its corners on the sides.

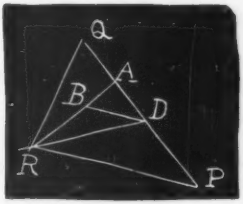
Also solved by J. Scheffer, and V. M. Spunar.

351. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Given an isosceles right triangle with hypothenuse h; an isosceles triangle with two sides h and two angles $A=22^\circ\ 30'$; a right angle triangle with the same angle A and opposite side $h/\sqrt{2}$; a triangle with the same angle A, opposite side h, and an angle 45°. Form a triangle whose four pieces are these four triangles, and prove geometrically that it is isosceles.

Solution by G. B. M. ZERR. A. M., Ph. D., Philadelphia, Pa.

Let ABD be the isosceles right triangle with hypothenuse $BD=h$, sides AB, $AD=h/\sqrt{2}$.



Produce AB to R, making $BR=BD$ and connect RD. Since $\angle ABD=45^\circ$, $\angle RBD=135^\circ$.

$\therefore\angle BRD=\angle RDB=22^\circ\ 30'=A$, and BRD is the isosceles triangle with sides h and opposite angles A. Take Q in DA produced so that $AQ=AD$ and join RQ. Then the right triangle ARQ has angle $ARQ=A$ and $AQ=h/\sqrt{2}$. Produce AD to P making $DP=h$ and join RP. Triangle DRP has angle $DRP=A$, $\angle P=45^\circ$, and side DP opposite $\angle DRP=h$.

Now $\angle PQR=67^\circ\ 30'=3A$, $\angle ARQ=\angle ARD=\angle DRP=A$.

$\therefore\angle QRP=3A$. $\therefore\angle Q=\angle QRP$, and the triangle PQR is isosceles.

CALCULUS.

281. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that if n be a positive integer greater than unity,

$$\text{Log}n=(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\ldots+\frac{1}{n-1}+\frac{1}{2n})-C+\frac{B_1}{2n^2}-\frac{B_2}{4n^4}+\frac{B_3}{6n^6}-\frac{B_4}{8n^8}+\ldots(1).$$

REMARK.—This development may be obtained by employing the formula given by the proposer in *Annals of Mathematics*, second series, Vol. 5, No. 4, July, 1904, but other proofs are also desired. It will be observed that (1) offers a ready method of evaluating C, which is remarkably simple and very rapidly convergent if $n>$ or $=10$. Compare with method given by Mr. Bromwitch in *Messenger of Mathematics*, October, 1906.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In Art. 304, Todhunter's *Integral Calculus*, we find the following result deduced:

$$\Sigma\phi(x)=C+\frac{1}{h}\int\phi(x)dx-\tfrac{1}{2}\phi(x)+B_1\phi'(x)\frac{h}{2}-B_2\phi'''(x)\frac{h^3}{4!}$$

$$+B_3\phi^{v}(x)\frac{h^5}{6!}-B_4\phi^{vii}(x)\frac{h^7}{8!}+\ldots$$

Let $h=1$, $\phi(x)=1/n$. Then $\phi'(x)=-1/n^2$, $\phi'''(x)=-3!/n^4$, $\phi^{v}(x)=-5!/n^6$, $\phi^{vii}(x)=-7!/n^8$.

$$\Sigma\phi(x)=1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{n-1}.$$

$$\therefore \Sigma\left(\frac{1}{n}\right)=C+\log n-\frac{1}{2n}-\frac{B_1}{2n^2}+\frac{B_2}{4n^4}-\frac{B_3}{6n^6}+\frac{B_4}{8n^8}-\ldots$$ whence the given result.

II. Solution by C. N. SCHMALL, New York City.

Take the well known relation

$$\Gamma(x+2)=(x+1)\Gamma(x+1)\ldots(1)$$

and take logarithms of both sides,

$$\log\Gamma(x+2)=\log\Gamma(x+1)+\log(x+1)\ldots(2).$$

Now define a function $\phi(x)$ by the relation

$$\phi(x)=\frac{d}{dx}\log\Gamma(1+x)\ldots(3).$$

Differentiating equation (2) we have

$$\phi(x+1)=\phi(x)+\frac{1}{x+1}\ldots(4).$$

$\therefore \phi(x)=\phi(x+1)-\frac{1}{x+1}$. Again, $\phi(x+2)=\phi[(x+1)+1]$, and by (4),

$$\phi[(x+1)+1]=\phi(x+1)+\frac{1}{(x+1)+1}=\phi(x+1)+\frac{1}{x+2}=\phi(x)+\frac{1}{x+1}+\frac{1}{x+2}$$

Transposing, $\phi(x)=\phi(x+2)-\frac{1}{x+1}-\frac{1}{x+2}$. Hence, by induction, we have

$$\phi(x)=\phi(x+n)-\left[\frac{1}{x+1}+\frac{1}{x+2}+\ldots+\frac{1}{x+n}\right]\ldots(5),$$

where n is an integer. Put $x=0$, then

$$\phi(n)=-C+1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{n}\ldots(6),$$

where $-C=\phi(0)$. Now (6) may be written

$$\phi(n)=-C+\int_0^1(1+z+z^2+\ldots+z^{n-1})dz=-C+\int_0^1\frac{(1-z^n)}{1-z}dz\ldots(7).$$

This defines $\phi(n)$ for integral values of n. Again, by (7),

$$\phi(n+1)-\phi(n)=\int_0^1\frac{(1-z^{n+1})\,dz}{1-z}-\int_0^1\frac{(1-z^n)dz}{1-z}=\int_0^1\frac{(z^n-z^{n+1})dz}{1-z}$$

$$=\int_0^1\frac{z^n(1-z)}{1-z}dz=\int_0^1 z^n dz=\frac{1}{n+1}.$$

Putting x for n, this becomes

$$\phi(x+1)-\phi(x)=\frac{1}{x+1}$$

which agrees with (4). Hence n may also be a fraction; and we have, in general, from (7),

$$\phi(x) = -C + \int_0^1 \frac{(1-z^x)}{1-z}\,dz = -C + \int_0^1 (1-z^x)(1+z+z^2+\ldots)\,dz$$

$$= -C + \frac{x}{x+1} + \tfrac{1}{2}\frac{x}{x+2} + \tfrac{1}{3}\frac{x}{x+3} + \ldots \quad (8).$$

Replace $\phi(x)$ by its value from (3), and integrate,

$$\log \Gamma(1+x) = -Cx + x - \log(1+x) + \tfrac{1}{2}x - \log(1+\tfrac{1}{2}x) + \tfrac{1}{3}x - \log(1+\tfrac{1}{3}x) + \ldots$$

Raise e to the power of each side,

$$\Gamma(1+x) = e^{-cx} \cdot \frac{e^x}{1+x} \cdot \frac{e^{\frac{1}{2}x}}{1+\frac{1}{2}x} \cdot \frac{e^{\frac{1}{3}x}}{1+\frac{1}{3}x} \ldots \quad (9).$$

Putting $x=1$, $1 = e^{-c} \cdot \tfrac{1}{2}e \cdot \dfrac{2e^{\frac{1}{2}}}{2} \cdot \dfrac{3e^{\frac{1}{3}}}{4} \cdot \dfrac{4e^{\frac{1}{4}}}{5} \ldots$ Taking logarithms,

$$0 = -C + 1 + \tfrac{1}{2} + \tfrac{1}{3} + \ldots + (1/n) - \log(n+1).$$

$\therefore \log(n+1) = -C + 1 + \frac{1}{2} + \frac{1}{3} + \ldots + (1/n) \equiv \phi(n)$ [by (6)], when n becomes infinite. Hence, for a very large value of n we have, approximately,

$$\phi(n) = \log(n+1) = \log n + \log(n+1) - \log n$$

$$= \log n + \log(1+\frac{1}{n}) = \log n + \frac{1}{n} - \frac{1}{2n^2} + \ldots.$$

Hence, $\log n = \log(n+1) - \dfrac{1}{n} + \dfrac{1}{2n^2} - \ldots + \ldots = -C + 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

$$+ \frac{1}{n-1} + \frac{1}{n} - \frac{1}{n} + \frac{1}{2n^2} - \ldots + \ldots$$

i. e., $\log n = (1 + \frac{1}{2} + \frac{1}{3} + \ldots + \dfrac{1}{n-1} - C + \dfrac{1}{2n^2} - \ldots + \ldots$

We may obtain C in the form of a definite integral, as follows:

$$\phi(x) = \frac{d}{dx}\log \Gamma(1+x) \text{ (by definition)} = \frac{d}{dx}\Gamma(1+x)/\Gamma(1+x).$$

$$\therefore \phi(x)\Gamma'(1+x) = \frac{d}{dx}\Gamma(1+x) = \frac{d}{dx}\int_0^\infty z^x e^{-z}dz = \int_0^\infty z^x e^{-z}\log z dz.$$

Put $x=0$; and since $\Gamma'(1)=1$, we have

$$\phi(0) = \int_0^\infty e^{-z}\log z dz. \qquad \therefore C = -\phi(0) = \int_0^\infty e^{-z}\log\left(\frac{1}{z}\right)dz$$

which gives Euler's constant.

MECHANICS.

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234. Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.

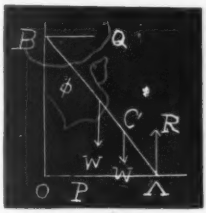
A ladder is placed with one end resting against a smooth wall and making with it an angle ϕ. Also, the roughness of the ground prevents it from slipping. A man weighing as much as the ladder ascends to the top. Taking μ as the coefficient of friction, prove:

(a) The ladder will slip before he gets to the top if $\phi > \tan^{-1} 4\mu/3$.

(b) If the ascent be feasible, there will be three times as much friction when he is at the top as when he is at the bottom, (See Jeans' *Theoretical Mechanics*, p. 47.)

I. Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let $AB=c$ be the ladder; let some position of the man be at C, AC being $=a$. Let the reaction at A and B be, respectively, $=P$ and Q, and the ladder be uniform, that is, its weight W acting at the mid-point of AB; the friction $P\mu$ acts along AO. Resolving along AO and BO, we have $Q=P\mu$, $P=2W$, and the equation of the moments with reference to A, $W(a+\frac{1}{2}c)\sin\phi = Qc\cos\phi = 2W\mu c\cos\phi$; whence $\tan\phi = \dfrac{4c\mu}{2a+c}$. For $a=c$, $\tan\phi = \dfrac{4\mu}{3}$, $Q=\frac{1}{2}W\tan\phi = \frac{2}{3}W\mu$. $P\mu = 2W\mu$.



II. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Let $2l$ be the length of the ladder, R the pressure on the ground, R' the pressure on the wall. Assuming that the man is at the top and that the ladder is about to slip, equating the sums of the vertical and horizontal forces to zero and taking moments about the foot of the ladder, we find

$$R=2W, \quad F=\mu R=R', \quad R'.2l\cos\phi = Wl\sin\phi + W.2l\sin\phi = 3Wl\sin\phi.$$

Whence, $2R'=3W\tan\phi$. But $R'=2\mu W$. Therefore, $4\mu=3\tan\phi$ or $\phi=\tan^{-1}(4\mu/3)$. A larger value of ϕ makes $R'>F$, and the man cannot reach the top before slipping begins.

Friction, F, when the man is at top$=R'=\frac{3}{2}W\tan\phi$. When the man is at the bottom, the moment equation gives $R'.2l\cos\phi=Wl\sin\phi$. Whence $R'=\frac{1}{2}W\tan\phi$.

Hence, friction when at the top, is three times as great as when at the bottom.

Solved similarly by G. B. M. Zerr.

PROBLEMS FOR SOLUTION.

ALGEBRA.

331. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Extract the square root of $21+6\sqrt{2}+2\sqrt{21}-6\sqrt{3}-6\sqrt{7}-2\sqrt{6}-2\sqrt{4}$ and also of $4\sqrt{2}+2\sqrt{6}-9-4\sqrt{3}$.

332. Proposed by C. N. SCHMALL, New York City.

Solve the quadratic, $x^2+ax+b=0$, without completing the square.

GEOMETRY.

359. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Two tangents are drawn to two confocal parabolas from any point on a common tangent. Show that the former two tangents and their chord of contact envelop yet another confocal parabola.

360. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A circular segment, area A, revolves successively about the diameters (fixed) d, d', intersecting at an angle θ. If $v=$volume about d, v' the volume about d', then $v^2+v'^2-2vv'\cos\theta$ is independent of the position of the segment.

361. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

$ABCD$ is a quadrilateral. The bisectors of A and C meet in O_1; those of B and D meet in O_2. Find the tangent of the angle between AD and O_1O_2 in terms of sines and cosines of A, D, $A+B$, and $A+D$.

CALCULUS.

389. Proposed by G. W. DROKE, Professor of Mathematics, University of Arkansas.

Find the curve such that the rectangle under the perpendiculars from two fixed points on the normals be constant.

MECHANICS.

242. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A weight W is supported by three strings of the same size and quality lying in the same plane. The middle string is vertical, one string makes with it an angle θ on one side, and the other string makes with it an angle ϕ on the other side. Find the stresses T_1, T_2, T_3 in the strings.

243. Proposed by C. N. SCHMALL, New York City.

In a game of billiards a player observes two balls, A and B, at rest in a certain position and concludes that it would be to his advantage to project A against B in such a manner that, as a result of the impact, A might suffer the greatest possible deviation from its course. Taking the balls to be equal and smooth, each of diameter a and elasticity e, and the distance between their centers to be d, show that he can accomplish the desired motion by projecting A in a line making an angle equal to the

$$\sin^{-1}\frac{a}{d}\sqrt{\frac{1-e}{3-e}}$$

with the line joining the centers.

NOTES AND NEWS.

In connection with the series of articles on the teaching of collegiate mathematics, the paper by Professor Smith in this number is most timely and suggestive. It is greatly to be desired that we in America should deliberately take account of stock mathematically, and nothing could give so great an incentive as this opportunity to join with the other important nations in both self inspection and mutual comparison of methods and results.

In the February number will appear the third paper of the series, by Professor W. A. Granville of Yale University, on "The Teaching of the Elements of Trigonometry." S.

The twenty-sixth meeting of the Chicago Section of the American Mathematical Society was held at Chicago on December 31, 1909, and January 1, 1910. There were in attendance forty-seven members of the Society and twenty-five papers were presented. Forty members dined together on New Year's Eve and enjoyed a most interesting and profitable occasion for the promotion of acquaintance and good fellowship. The officers of the Section elected for 1910 are: Professor L. E. Dickson, chairman; Professor H. L. Slaught, secretary; and Professor W. B. Ford, third member of the program committee. Professor G. A. Miller has been chairman of the Section for the past two years, and was also secretary of Section A of the

American Association for the Advancement of Science. Professor Miller attended both the Boston and Chicago meetings and presented papers before both meetings. S.

The American Mathematical Society held its annual meeting at Boston during holiday week in affiliation with the American Association. Joint sessions were held with the Physics and Mathematics Sections and a joint dinner brought together the representatives of these sections with the New England Association of Mathematic Teachers. Eighteen papers were read before the sessions of the American Mathematical Society and seven before the joint session. S.

Professor E. H. Moore, University of Chicago, has been elected chairman of Section A of the American Association for the Advancement of Science for the meeting to be held at the University of Minnesota during the next convocation week. M.

Professor E. J. Wilczynski, University of Illinois, has recently been awarded a prize by the Belgian Royal Academy. This would appear to be the second prize in pure mathematics awarded to an American by a foreign Academy. M.

BOOKS AND PERIODICALS.

Plane Geometry Developed by the Syllabus Method. By Eugene Randolph Smith, A. M., Head of the Department of Mathematics, Polytechnic Preparatory School, Brooklyn, N. Y. (formerly Head of the Department of Mathematics, Montclair High School). 8vo. Cloth. 192 pages. New York and Chicago: American Book Co.

The object of this book seems to be to meet the demands of those teachers who wish only the skeleton of a demonstration, leaving the work of putting on the muscles and giving form and symmetry to the geometric body, to the student. Those teachers, too, who wish no text will find this book suited to their needs.

The book opens with a section on Logic. This feature of the book will be condemned by those teachers of geometry who believe that Euclidean geometry should be as much divorced from formal logic as are the other branches of mathematics.

The exercises are in two divisions, those under the theorems chosen to illustrate the use of them and those in general lists, to give the student practice in finding for himself the underlying principle.

The book will be found very suggestive for teachers of geometry. F.

Plane Trigonometry. By Edward R. Robbins, Senior Mathematical Master, The William Penn School. 8vo. Cloth. 153 pages. New York and Chicago: The American Book Co.

This contains only the most essential facts and principles of the subject. F.

A Course in Mathematics. For Students of Engineering and Applied Science. By Frederick S. Woods and Frederick H. Bailey, Professors of Mathematics in the Massachusetts Institute of Technology. Vol. II, Integral Calculus, Functions of Several Variables, Space Geometry, Differential Equations. 8vo. Cloth. xi+410 pages. Price, $2.25. Boston and Chicago. Ginn & Co.

This volume completes the plan of the course of study in mathematics outlined in the preface of the first volume. Integration of functions of a single variable is treated in the first eight chapters, emphasis being laid upon fundamental processes. In this part of the book the authors have added what seems to them to be a new feature, viz., the treatment of simple differential equations in close connection with integration long before the formal study of such equations.

With the ninth chapter begins the study of functions of two or more variables, being accompanied by the use of solid analytical geometry, and the treatment of partial differentiation and of multiple integrals. Also here is introduced a short discussion of line and surface integrals.

The latter part of the book consists of chapters on series,—Fourier, Taylor, etc.,—complex numbers and differential equations.

This, with the companion first volume, will form an excellent course in mathematics for the practical mathematician and will give the teacher of mathematics who does not wish to teach the subject in closed compartments a good outline for that kind of work. F.

Strength of Materials. An Elementary Study Prepared for the Use of Midshipmen at the U. S. Naval Academy. By H. E. Smith, Professor of Mathematics, U. S. Navy. 12mo. Cloth. ix+170 pages, 73 figures. Price, $1.25. New York: John Wiley & Sons.

The student of applied mathematics will find in this little volume a fine presentation of the subject. While the treatment is necessarily brief, yet it is clear. The problems are numerous and well selected. Teachers having only a limited time to devote to the subject, will do well to examine this book with a view to its use in their classes. F.

In Starland with a Three-Inch Telescope. By William Taylor Olcott, Author of a "Field Book of the Stars." 12mo. Cloth. xiv+146 pages. Price, $1.00. New York and London: G. P. Putnam's Sons.

This little volume will be found to be of inestimable value to teachers and students having the use of a small telescope. The constellations are grouped with the seasons for the sake of convenience and in connection with the forty groups diagramed is given a brief description of the most interesting objects to be seen in each group. F.

Elements of Plane and Spherical Trigonometry. By James Howard Gore, Ph. D., Professor of Mathematics, George Washington University, Author of Plane and Spherical Geometry, Elements of Geodesy, History of Geodesy, Bibliography of Geodesy, etc., etc. With six-place tables of logarithms of numbers from 1 to 10000 and logarithmic sines, cosines, tangents and cotangents from 0° to 90°, and an auxiliary table for small angles. 8vo. Cloth. vi+122 pages, +xvii+80 pages of tables. Price, $1.25. New York and London: G. P. Putnam's Sons.

This book does not differ essentially from many of the recent texts on the subject.

F.

The Calculus and Its Applications. A Practical Treatise for Beginners, Especially Engineering Students. By Robert Gordon Blaine, M. E., Assoc. M. Inst., C. E., etc., Lecturer at the City Guild's Technical College, Finsbury, London, E. C., Author of Hydraulic Machinery, Lessons in Practical Mechanics, The Slide Rule, etc. 8vo. Cloth. ix+321 pages. Price, $1.50. New York: D. Van Nostrand & Co.

This is a good text, well written, containing numerous interesting problems, suitable for the student of pure mathematics as well as the student of applied mathematics.

The fundamental principles of the Calculus are established in a practical way and will be easily understood by the student. F.

Plane and Spherical Trigonometry. By Levi L. Conant, Ph. D., Professor of Mathematics in the Worcester Polytechaic Institute. 8vo. Cloth. 222 pages, +80 pages of tables. New York & Chicago: American Book Co.

In addition to the usual matter to be fou..d in a text of this sort, the author has added a chapter on hyperbolic functions. The usual topics are treated with clearness and rigor. F.

Five-Place Logarithmic and Trigonometric Tables. Edited by James M. Taylor, Colgate University. 8vo. Cloth. xv+72 pages. Boston and Chicago: Ginn & Co.

A convenient feature of these tables is to be found in the patent index, greatly facilitating the work of finding the logarithms of numbers and functions of angles. F.

Applications of the Calculus to Mechanics. By E. R. Hedrick, Professor of Mathematics, University of Missouri, and O. D. Kellogg, Assistant Professor of Mathematics, University of Missouri. 8vo. Cloth. vi+116 pages. Boston and Chicago: Ginn & Co.

This book is a formulation of the work done in the Missouri State University following the course in Sophomore Calculus. It is believed that the work here outlined and pursued by the student will fix the principles of the Calculus more firmly in mind. A number of problems of some difficulty are inserted in each chapter. F.

The Fundamental Principles of Chemistry. An Introduction to all Text-Books of Chemistry. By William Ostwald. Authorized Translation by Harry W. Morse. 8vo. Cloth. xii+349 pages, with 65 figures in the text. Price, $2.25. New York: Longman's, Green & Co.

The author states in the preface that the object of this book is to present the fundamental principles of Chemistry, their meaning and connection free from irrelevant additions, and the book represents an opinion of the author that it is possible to work out a chemistry in the form of a rational scientific system without bringing in the properties of individual substances. The work is one of very great interest to the teacher of Physics as well as of Chemistry. F.

The Monist. A Quarterly Magazine devoted to the Philosophy of Science. Edited by Paul Carus. Price, $2.00 per year in advance.

The January, 1910, number contains three articles of much interest to mathematicians. The first is The Nature of Logical and Mathematical Thought, by Paul Carus; the second, The Future of Mathematics, by H. Poincare, translated by Dr. Halsted; and the third, Transfinite Numbers and the Principles of Mathematics, by Philip E. B. Jourdain. F.

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THE AMERICAN MATHEMATICAL MONTHLY.

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ON THE TEACHING OF THE ELEMENTS OF PLANE TRIGONOMETRY.

By W. A. GRANVILLE, Yale University.

In teaching we proceed from that which is familiar to that which is new. The successful teacher leads the student into a knowledge of Trigonometry by making the successive steps to it as gradual and natural as possible. From his knowledge of Geometry the student has usually a fair understanding of the relations between the sides and angles of a triangle; and of the different kinds of triangles, he is most familiar with the right triangle. Hence it is best to commence with the trigonometric functions of an acute angle and to define them as the ratios of the sides of a right triangle having that acute angle as one of its angles. The functions should be defined and written in pairs as reciprocals of each other in order to aid the memory and to emphasize one of the most important of their functional relations. No attempt should be made at the start to teach the student any particular definition of angle; let angles be to him just what they were to him in his Geometry. The general definition of angle should not be given until the student has mastered the right triangle. Nor is it necessary at this time to introduce circular measure (radians). Only even degrees should be given at first, then minutes or decimal parts of a degree. The use of seconds to to any considerable extent in a first course in Trigonometry has a tendency to obscure the theory, and the additional calculations involved are apt to degenerate into mere drudgery for student and teacher alike. The division of the degree into decimal parts, instead of using minutes and seconds, has much favor by expert computers. Irrespective of what the future of the Metric System may be in the United States, it seems certain that the decimal division of the degree is fast gaining ground in both theoretical and practical work. And right here I wish to record a most emphatic protest against a notation in which 36.2° is written $36^\circ.2$. There is no reason whatever for inserting the unit of measurement between the digits of a number. What would we think of the engineer who wrote 127.ft.36 instead of 127.36